Matching Under Preferences: Fairness - III

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Pallavi Jain
Department of Computer Science and Engineering
IIT Jodhpur

Fairness Notions

- Egalitarian Stable Matching
- Min Regret Stable Matching
- Sex-Equal Stable Matching
- Balanced Stable Matching
- Median Stable Matching
- Large Size
- Popular Matching
- Popularity with less blocking edges
- Many More...

μ - a stable matching

Min Regret Stable Matching

$$egal(\mu) = \sum_{xy \in \mu} \big(r(x,y) + r(y,x)\big) + \sum_{x \notin V(\mu)} (|\text{pref}(x)| + 1)$$

$$regret(\mu) = \max\{\max_{xy \in \mu} r(x,y), \max_{x \notin V(\mu)} |(\text{pref}(x)| + 1)\}$$

Can we find minimum regret stable matching in polynomial time?

A: X Y Z W

regret cost =2

A can't be matched with Z and W

A: X Y

find a **stable matching** in the reduced instance.

Sex-Equal Stable Matching

Input: an instance of stable marriage problem, and a positive integer η

Question: Does there exist a stable matching μ such that

$$\left| \sum_{mw \in \mu} mr(m, w) - \sum_{mw \in \mu} wr(w, m) \right| \leq \eta$$

NP-complete — even when $\eta = 0$ and length of preference list is at most 3

$\mathcal{O}(n^3)$ — when the length of preference list on one side is at most 2

McDermid and Irving, Sex-equal stable matchings: Complexity and exact algorithms

Balanced Stable Matching

Input: an instance of stable marriage problem, and a positive integer η

Question: Does there exist a stable matching μ such that

$$\sum_{mw \in \mu} mr(m, w) \leq \eta \quad \text{and} \quad \sum_{mw \in \mu} wr(w, m) \leq \eta$$

NP-complete

There exists algorithms with running time $f(\eta) \cdot n^{\mathcal{O}(1)}$

Median Stable Matching

- match to the middle most person among all partners
- Z = set of all stable matchings in a given instance $\mathscr{I}$
- For each man m, sort the multiset of women $\{\mu(m) : \mu \in Z\}$ from m's most preferred to least preferred woman.
- StablePartners(m) be this sorted set
- α^M_{median} is a matching obtained by matching every man m to the median in StablePartners(m)
- α^M_{median} is a stable matching
- We can do the same for woman ...

Size vs Stability

Almost Stable Matching

- size of every stable matching is same
- under utilisation of the resources
- If we want to increase the size, we need to allow some blocking edges

Input: an instance of the stable marriage problem, and two integers k, t

Question: Does there exist a matching whose size is at least t and the number of blocking edges is at most k?

Popular Matching

- find a matching that is preferred by *more* agents.
- find a matching μ such that there is no matching μ' such that

$$\text{the number agents preferring } \mu' \geq \text{the number agents preferring } \mu$$

Thank You!